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III

16992743

Total No. of Questions - 24

Total No. of Printed Pages - 3

Regd. No.

Part -III

MATHEMATICS, Paper - I (A)

(English Version)

Time : 3 Hours **Max. Marks : 75**

Note: This question paper consists of THREE Sections - A, B and C.

SECTION - A

I. Very short answer type questions - **10×2=20**

(i) Answer **ALL** the questions.

(ii) Each question carries **TWO** marks.

1. Find the domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$.

2. If $A = \{1, 2, 3, 4\}$ and $f : A \to R$ is a function defined by $f(x) = \frac{x^2-x+1}{x+1}$, then find the range of f.

3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$, then find the value of k.

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.

5. OABC is a parallelogram. If $\overline{OA} = \bar{a}$ and $\overline{OC} = \bar{c}$, find the vector equation of the side BC.

6. Find a vector in the direction of vector $\bar{a} = \bar{i} - 2\bar{j}$ that has magnitude 7 units.

7. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p.

8. Prove that $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \cot 36°$.

9. Find a sine function whose period is $\frac{2}{3}$.

10. For any $x \in R$, prove that $\cos h^4 x - \sin h^4 x = \cos h\,(2x)$.

SECTION B

II. Short answer type questions - **5×4=20**

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. Find the adjoint and the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$.

12. Show that the points $A(2\bar{i} - \bar{j} + \bar{k})$, $B(\bar{i} - 3\bar{j} - 5\bar{k})$, $C(3\bar{i} - 4\bar{j} - 4\bar{k})$ are the vertices of a right angled triangle.

13. Let $\bar{e}_1$ and $\bar{e}_2$ be unit vectors making angle θ. If $\frac{1}{2}|\bar{e}_1 - \bar{e}_2| = \sin(\lambda\theta)$, then find λ.

14. Show that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$.

15. Solve $1 + \sin^2\theta = 3\sin\theta\cos\theta$.

16. Prove that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(2\tan^{-1}\frac{3}{4}\right)$.

17. If $a = (b - c)\sec\theta$, prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$.

SECTION - C

III. Long answer type questions - **5×7=35**

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Let f: A $\to$B, g: B $\to$ C be bijections, then prove that $(gof)^{-1} = f^{-1} og^{-1}$.

19. Using mathematical induction, prove that $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots\ldots$ upto n terms $= \frac{n(n+1)^2(n+2)}{12}$, for all $n \in N$.

20. Solve the following system of equation by using Cramer's rule.

$x + y + z = 9$

$2x + 5y + 7z = 52$

$2x + y - z = 0$

21. Show that -

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

22. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}, \bar{b} = \bar{i} + \bar{j} - \bar{k}$, and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$.

23. If A, B, C are angles in a triangle, then prove that $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$.

24. In a ΔABC, if a = 13, b = 14, c = 15, show that $R = \frac{65}{8}$, r = 4, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.

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